Simulación y optimización de procesos

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Optimización de funciones utilizando MATLAB & GAMS

I. Rutinas para optimización en MATLAB

Nonlinear minimization of functions.

fminbnd - Scalar bounded nonlinear function minimization.
fmincon - Multidimensional constrained nonlinear minimization.
fminsearch - Multidimensional unconstrained nonlinear minimization, by Nelder-Mead direct search method.
fminunc - Multidimensional unconstrained nonlinear minimization.
fseminf - Multidimensional constrained minimization, semi-infinite constraints.

Nonlinear minimization of multi-objective functions.

fgoalattain - Multidimensional goal attainment optimization
fminimax - Multidimensional minimax optimization.

Linear least squares (of matrix problems).

lsqlin - Linear least squares with linear constraints.
lsqnonneg - Linear least squares with nonnegativity constraints.

Nonlinear least squares (of functions).

lsqcurvefit - Nonlinear curvefitting via least squares (with bounds).
lsqnonlin - Nonlinear least squares with upper and lower bounds.

Nonlinear zero finding (equation solving).

fzero - Scalar nonlinear zero finding.
fsolve - Nonlinear system of equations solve (function solve).

Minimization of matrix problems.

linprog - Linear programming.
quadprog - Quadratic programming.

Controlling defaults and options.

optimset - Create or alter optimization OPTIONS structure.
optimget - Get optimization parameters from OPTIONS structure.

optimset

DerivativeCheck: [on | {off}]
Diagnostics: [on | {off}]
DiffMaxChange: [positive scalar {1e-1}]
DiffMinChange: [positive scalar {1e-8}]
Display: [off | iter | {final}]
GoalsExactAchieve: [positive scalar | {0}]
GradConstr: [on | {off}]
GradObj: [on | {off}]
Hessian: [on | {off}]
HessPattern: [sparse matrix]
HessUpdate: [dfp | gillmurray | steepdesc | {bfgs}]
JacobPattern: [sparse matrix]
Jacobian: [on | {off}]
LargeScale: [{on} | off]
LevenbergMarquardt: [on | off]
LineSearchType: [cubicpoly | {quadcubic}]
MaxFunEvals: [positive scalar]
MaxIter: [positive scalar]
MaxPCGIter: [positive scalar]
MeritFunction: [singleobj | multiobj]
MinAbsMax: [positive scalar | {0}]
PrecondBandWidth: [positive scalar | Inf]
TolCon: [positive scalar]
TolFun: [positive scalar]
TolPCG: [positive scalar]
TolX: [positive scalar]
TypicalX: [vector]

II Ejemplo
Codificación en MATLAB para resolver el problema de Rosenbrock

```
% inico del programa **** optimo.m  *****
% programa de prueba para las rutinas de optimizacion
% aplicado al problema de Rosenbrock
% f(x1,x2)= 100*(x2-x1^2)^2+(1-x1)^2;

% con la solucion siguiente
%
******************************************************************
****************
% X = 0.99999562761312   0.99999125498266   ****  es el punto
optimo ****          *
% FVAL = 1.911777389766082e-011   ***** la funcion evaluada den el
optimo          *
% EXITFLAG =  1  **** el metodo convergio correctamente
*
% OUTPUT =  iterations: 26
*
%           funcCount: 162
*
%           stepsize: 1.29917460625632
*
%           firstorderopt: 5.002279231295639e-004
*
%           algorithm: 'medium-scale: Quasi-Newton line search'
*
% GRAD =  1.0e-003 *   -0.50022792312956
*
%                      -0.18877516066886
*
% HESSIAN =  1.0e+002 *    8.20403078550788  -4.09549747125395
*
%                         -4.09549747125395   2.04772027690417
*
% valores_propios =   1.0e+003 *  1.02491693258887
*
%                                 0.00025817365234
*
%
******************************************************************
***************

clc;clear all; format compact;  format long;
% estimado inicial
x0=[-1.2 1.0];
% calculo del optimo
fprintf(' ************* resumen de resultados
*************************  \n')
[X,FVAL,EXITFLAG,OUTPUT,GRAD,HESSIAN]=FMINUNC('FUN',x0)
```

```
% matriz hessiana
H=HESSIAN;
valores_propios=eig(H)
% fin del archivo  ****** optimo.m *****
```

```
% rutina de declaracion de la funcion objetivo
function f=FUN(x)
x1=x(1); x2=x(2);
f(1)= 100*(x2-x1^2)^2+(1-x1)^2;
% fin de la rutina
```

III Uso de GAMS

The General Algebraic Modeling System (GAMS) es un sistema de modelado de alto nivel para problemas de programación matemática, consiste de un compilador de lenguaje y un conjunto de rutinas de optimización como son: DICOPT, SCICONIC, CPLEX, MINOS

a. **Ejemplo de codificación para la solucion del problema 8.26 del libro de Himmelblau (2001)**

```
**********************************************************************
$ TITLE  problema 8 26
˝
$ OFFSYMXREF
˝
$ OFFSYMLIST
˝

˝
OPTION LIMROW=0;
˝
OPTION LIMCOL=0;
˝

˝
* ejemplo para resolver el problema 8 26 del libro
˝
* declaracion de las variables del problema
positive variables x1, x2, x3 ;

* declaracion de la varibale de la funcion objetivo
free variable f;

* delcaracion del conjunto de ecuaciones del modelo
equations  obj, g1, g2, g3, h1, h2, h3;

* declaracion de las expresiones de las ecuaciones
obj.. f =e= x1**2 + x2 **2 + x3 **2;
g1..  -2*x1 - x2 =g= -5;
```

```
g2 .. -x1 - x3 =g= -2;
g3..  -x1 - 2*x2 - x3 =g= -10;
h1..  2*x1 - 2*x2 + x3 =e= -2;
h2..  10*x1 + 8*x2 - 14*x3 =e= 26;
h3..  -4*x1 + 5*x2 - 6*x3 =e= 6;

* declaracion de los limites de las variables

x1.l = 1.;
x2.l = 2.;
x3.l = 0.;

* asignacion del nombre al problema
model p826  /all/;

* declaracion de solucion del problema

solve p826 using NLP minimizing f;

* fin de la codificacion del archivo p826 gms
```

---

b. Para el cual se obtuvieron los resultados siguientes en el archivo p826.lst

```
GAMS 2.25  PC AT/XT                              11/14/02 12:37:27  PAGE
1
˝
problema 8 26
˝

˝

˝

˝

˝
OPTION LIMROW=0;
˝
OPTION LIMCOL=0;

* ejemplo para resolver el problema 8 26 del libro
* declaracion de las variables del problema
positive variables x1, x2, x3 ;

* declaracion de la varibale de la funcion objetivo
free variable f;

* delcaracion del conjunto de ecuaciones del modelo
equations  obj, g1, g2, g3, h1, h2, h3;

* declaracion de las expresiones de las ecuaciones
obj.. f =e= x1**2 + x2 **2 + x3 **2;
g1..  -2*x1 - x2 =g= -5;
g2 .. -x1 - x3 =g= -2;
g3..  -x1 - 2*x2 - x3 =g= -10;
```

```
  23  h1..  2*x1 - 2*x2 + x3 =e= -2;
  24  h2..  10*x1 + 8*x2 - 14*x3 =e= 26;
  25  h3..  -4*x1 + 5*x2 - 6*x3 =e= 6;
  26
  27  * declaracion de los limites de las variables
  28
  29  x1.l = 1.;
  30  x2.l = 2.;
  31  x3.l = 0.;
  32
  33  * asignacion del nombre al problema
  34  model p826  /all/;
  35
  36  * declaracion de solucion del problema
  37
  38  solve p826 using NLP minimizing f;
  39
  40  * fin de la codificacion del archivo p826 gms
  41
  42


COMPILATION TIME     =        0.000 SECONDS        VERID TP5-00-038
GAMS 2.25  PC AT/XT                              11/14/02 12:37:27  PAGE
2
problema 8 26
Model Statistics    SOLVE P826 USING NLP FROM LINE 38




MODEL STATISTICS

BLOCKS OF EQUATIONS       7     SINGLE EQUATIONS        7
BLOCKS OF VARIABLES       4     SINGLE VARIABLES        4
NON ZERO ELEMENTS        20     NON LINEAR N-Z          3
DERIVATIVE POOL           6     CONSTANT POOL           1
CODE LENGTH              30

GENERATION TIME      =        0.050 SECONDS


EXECUTION TIME       =        0.110 SECONDS        VERID TP5-00-038
GAMS 2.25  PC AT/XT                              11/14/02 12:37:27  PAGE
3
problema 8 26
Solution Report     SOLVE P826 USING NLP FROM LINE 38


               S O L V E      S U M M A R Y

     MODEL   P826                OBJECTIVE  F
     TYPE    NLP                 DIRECTION  MINIMIZE
     SOLVER  MINOS5              FROM LINE  38

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
```

```
**** OBJECTIVE VALUE                5.0000

 RESOURCE USAGE, LIMIT          0.551     1000.000
 ITERATION COUNT, LIMIT         2         1000
 EVALUATION ERRORS              0            0

     M I N O S    5.3    (Nov 1990)         Ver: 225-DOS-02
     = = = = =

     B. A. Murtagh, University of New South Wales
       and
     P. E. Gill,  W. Murray,  M. A. Saunders and M. H. Wright
     Systems Optimization Laboratory, Stanford University.

 D E M O N S T R A T I O N    M O D E
   You do not have a full license for this program.
   The following size retrictions apply:
      Total nonzero elements:  1000
      Nonlinear nonzero elements:  300


 Estimate work space needed    --     39 Kb
 Work space allocated          --    147 Kb



 EXIT -- OPTIMAL SOLUTION FOUND
 MAJOR ITNS, LIMIT             1     200
 FUNOBJ, FUNCON CALLS          6       0
 SUPERBASICS                   0
 INTERPRETER USAGE           .00
 NORM RG / NORM PI     0.000E+00

                       LOWER     LEVEL     UPPER    MARGINAL

---- EQU OBJ             .         .         .        1.000
---- EQU G1            -5.000    -4.000     +INF       .
---- EQU G2            -2.000    -1.000     +INF       .
---- EQU G3           -10.000    -5.000     +INF       .
---- EQU H1            -2.000    -2.000    -2.000    -0.667
---- EQU H2            26.000    26.000    26.000     0.333
---- EQU H3             6.000     6.000     6.000      .

                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR X1              .        1.000     +INF       .
---- VAR X2              .        2.000     +INF       .
---- VAR X3              .         .        +INF      5.333
---- VAR F             -INF       5.000     +INF       .
```

```
**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS


EXECUTION TIME       =        0.220 SECONDS        VERID TP5-00-038


USER: CACHE DESIGN CASE STUDIES SERIES              G911007-1447AX-TP5
      GAMS DEMONSTRATION VERSION

**** FILE SUMMARY

INPUT      C:\GAMS2\P826.GMS
OUTPUT     C:\GAMS2\P826.LST
```

IV Ejercicios

1. Por medio del uso de MATLAB & GAMS calcule

a. min $f(x_1, x_2) = x_1^3 \exp(x_2-x_1^2-10(x_1-x_2)^2)$ con $x_0^T = [1\ 1]$

b. opt $f(u_1, u_2, u_3) = u_1^2 + u_2^2 + u_3^2$

donde

$u_1 = 1.5 - x_1 (1 - x_2)$

$u_2 = 2.25 - x_1 (1 - x_2^2)$

$u_3 = 2.625 - x_1 (1 - x_2^3)$

c. min $f(x1, x2, x3) = -(x_1^2 + x_2^2 + x_3^2)$

sujeta a

$x_1 + 2\ x_2 + 3\ x_3\ -1 = 0$

$x_1^2 + x_2^2\ /2 + x_3^2\ / 2 - 4 = 0$

$x_0^T = [2\ 2\ 2]$

compare sus resultados obtenidos