CBE495 LECTURE IV
MODEL PREDICTIVE CONTROL

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Spring 2006

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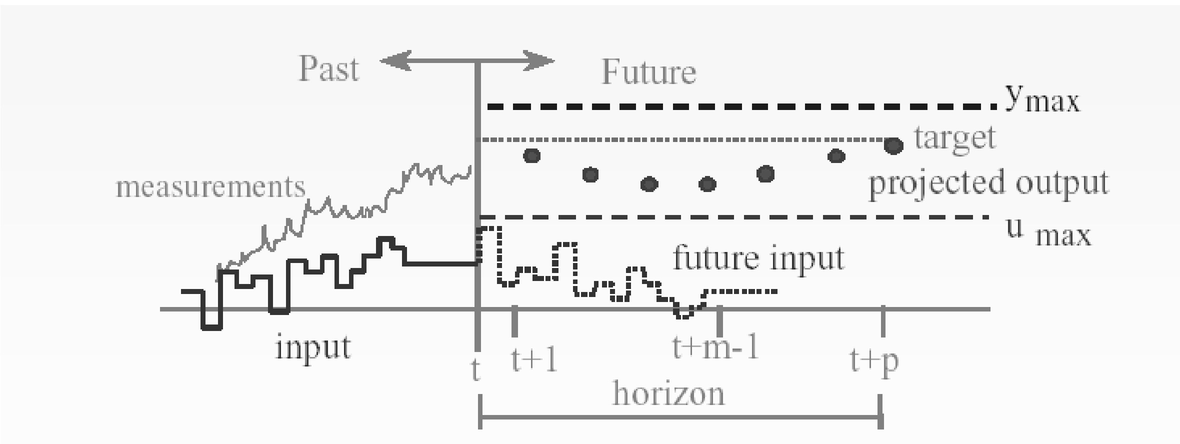
* Some parts are from Jay H. Lee's lecture notes

What is Model Predictive Control (MPC)?

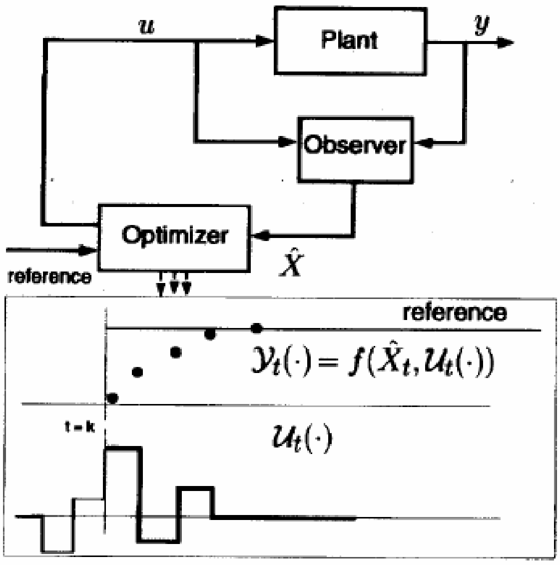
- **Multivariable control**
  - Calculate all the MV's at the same time based on all PV values
  - Not like multiloop control, no decoupling scheme is needed
  - More complicated
- **Constraints handling**
  - Process industry requires many constraints
    - Safety
    - Operational limitations
    - Product quality
  - No previous methodology handles constraints explicitly
- **Flexible formulation**
  - Many control objectives can be formulated as the objective function and constraints

- # Features
  - **Computer control:** sampled-data control
  - **Model-based control:** dynamic model is required
    - **Fundamental model**
    - **Empirical model (usually step response based)**
  - **Predictive:** adjust process based on the future prediction
    - **Not just based on the current error**
  - **Optimization-based:** no explicit control law
    - **Formulated with objective function and constraints**
    - **Optimization is solved at each sampling time**
  - **Integrated:** constraints and economic handling
    - **Optimizing control**
    - **Servo or regulatory control**
  - **Receding horizon control:** future window is moving forward
    - **Repeat the prediction and optimization at each sample time**
    - **Update the input based on the new measurement**

- # Similarity to human decision-making
  - **Sense:** collect new information
  - **Assess:** update the memories
  - **Predict:** forecast the outcome for a variety of possible decisions
  - **Optimize:** determine the best decision for given objective and constraints
  - **Implement:** the action for this time is imposed
  - **Repeat:** information collection, update and optimization done every so often



- **Exemplary Algorithm**



$$\min_{\mathcal{U}_t(\cdot)} \int_t^{t+p} l_1[\mathrm{Error}(\tau)] + l_2[\mathrm{Input}(\tau)]d\tau$$ ⟶ **Objective function**

$$\mathcal{U}(\cdot) \in U, \;\; \mathcal{Y}_t(\cdot) \in Y$$ ⟶ **Constraints**

Industrial Use of MPC

- **Initiated at Shell Oil and other refineries during late 70s and early 80s.**
- **Various commercial software**
  - **DMCplus – Aspen Tech**
  - **RMPCT–Honeywell**
  - **Dozen+ other players (e.g., 3DMPC-ABB)**
  - **>3000 worldwide installations**
- **Predominantly in the oil and petrochemical industries but the range of applications is expanding.**
- **Models used are predominantly empirical models developed through plant testing.**
- **Technology is used not only for multivariable control, but for most economic operation within constraint boundaries.**

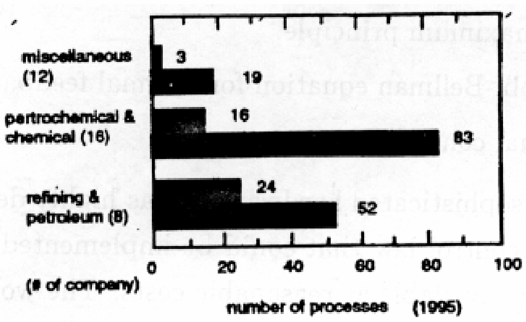
Survey Result (1)

- **Application by 5 major MPC vendors in North America / Europe (Badgwell, 1996)**

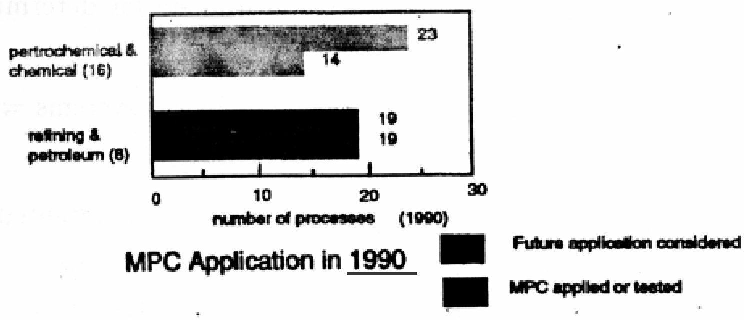
Area	DMC Coop.	Setpoint Inc.	Honeywell Profimatics	Adersa	Treiber Controls	Total
Refining	360	320	290	280	250	1500
Petrochemicals	210	40	40	-	-	290
Chemicals	10	20	10	3	150	193
Pulp and Paper	10	-	30	-	5	45
Gas	-	-	5	-	-	5
Utility	-	-	2	-	-	2
Air Separation	-	-	-	-	5	5
Mining/Metallurgy	-	2	-	7	6	15
Food Processing	-	-	-	41	-	41
Furnaces	-	-	-	42	-	42
Aerospace/Defence	-	-	-	13	-	13
Automotive	-	-	-	7	-	7
Other	10	20	-	45	-	75
Total	600	402	377	438	416	2233
First App	DMC:1985	IDCOM-M:1987 SMCA:1993	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	OPC:1987	
Largest App	603×283	35×28	28 ×20	-	24×19	

Survey Result (2)

- **Applications in Japan (Oshima, 1995)**



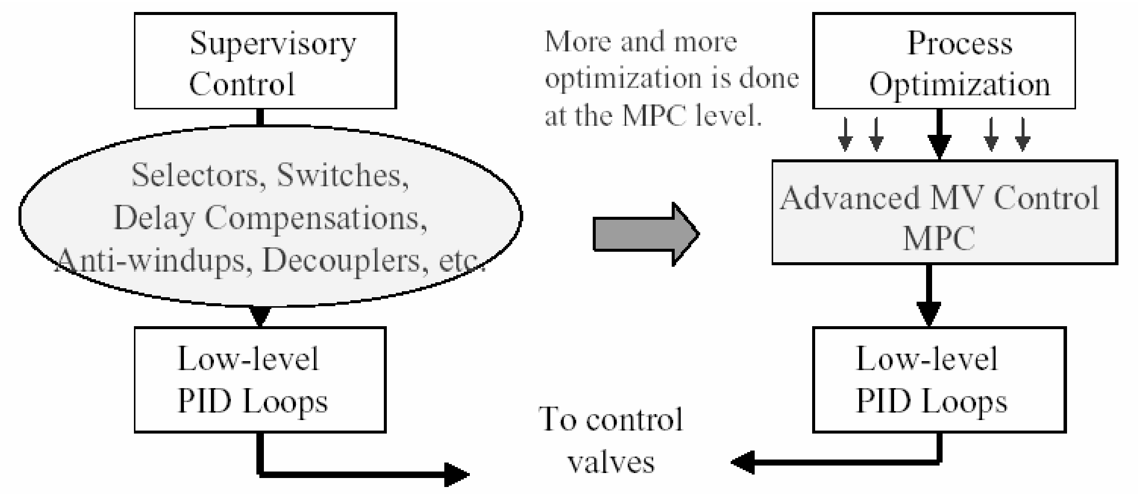
MPC Application in 1995



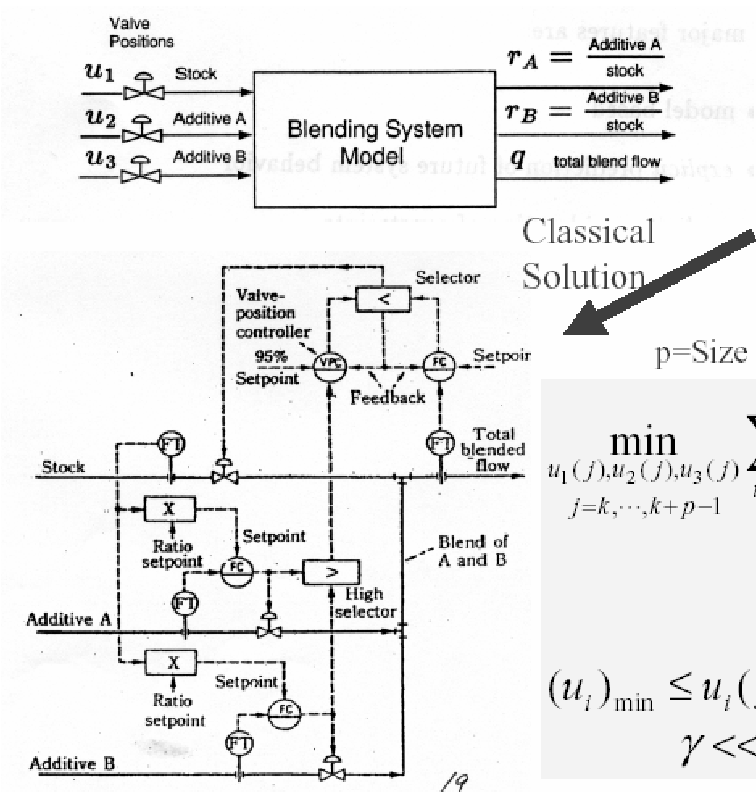
MPC Application in 1990

Reason for Popularity (1)

- **MPC provides a systematic, consistent, and integrated solution to process control problems with complex features**:
  - **Delays, inverse responses and other complex dynamics.**
  - **Strong interactions (e.g., large RGA)**
  - **Constraints (e.g., actuator limits, output limits)**



- **Example 1: Blending control system**



- Control r_A and r_B.
- Control q if possible.
- Flowrates of additives are limited.

Classical Solution

MPC: Solve at each time k

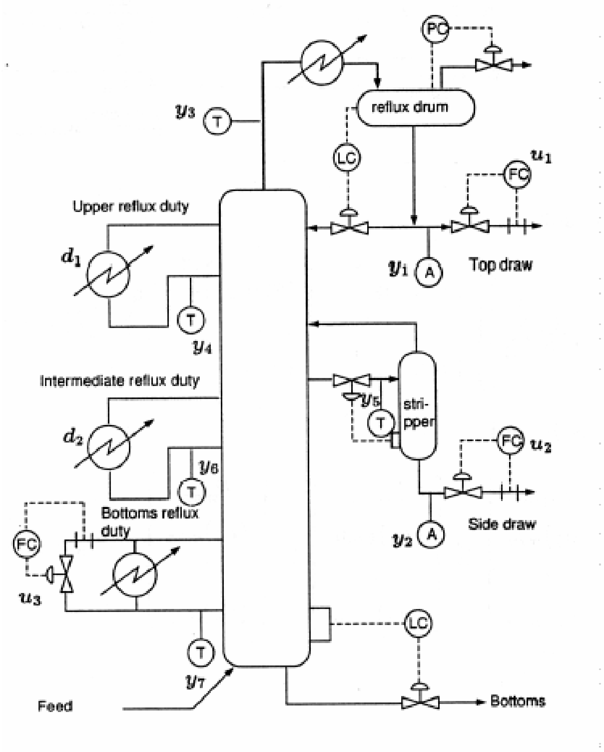
p=Size of prediction window

$$\min_{\substack{u_1(j),u_2(j),u_3(j) \\ j=k,\cdots,k+p-1}} \sum_{i=1}^{p} \left(r_A(k+i\,|\,k) - r_A{}^*\right)^2 + \left(r_B(k+i\,|\,k) - r_B{}^*\right)^2 + \gamma\left(q(k+i\,|\,k) - q^*\right)^2$$

$$(u_i)_{\min} \le u_i(j) \le (u_i)_{\max}, i = 1,\cdots,3,$$

$$\gamma << 1$$

• Example 2: Heavy Oil Fractionator



- Keep $y_7 \geq T_{min}$
- Control the two compositions y_1 and y_2
- Minmize u_3 to maximize the heat recovery.

Solution using the classical tools will be very complicated and a satisfactory solution is not known.

It is fairly easy to translate the above objective (as well as the valve limits) as a minimization function and inequality constraints as required by MPC.

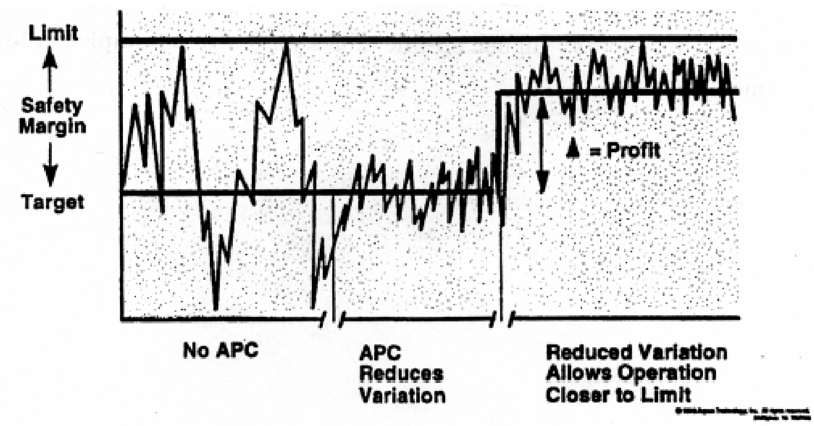
• Advantages of MPC over Traditional APC

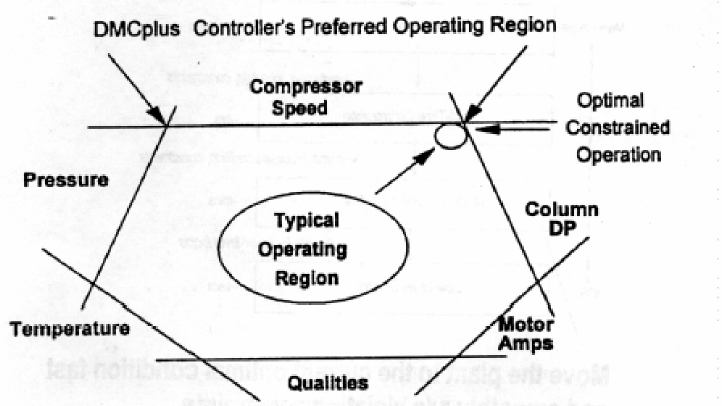
- **Integrated solution**
  - **automatic constraint handling**
  - **Feedforward/feedback**
  - **No need for decoupling or delay compensation**
- **Efficient Utilization of degrees of freedom**
  - **Can handle nonsquare systems (e.g., more MVs and CVs)**
  - **Assignable priorities, ideal settling values for MVs**
- **Consistent, systematic methodology**
- **Realized benefits**
  - **Higher on-line times**
  - **Cheaper implementation**
  - **Easier maintenance**

Reason for Popularity (2)

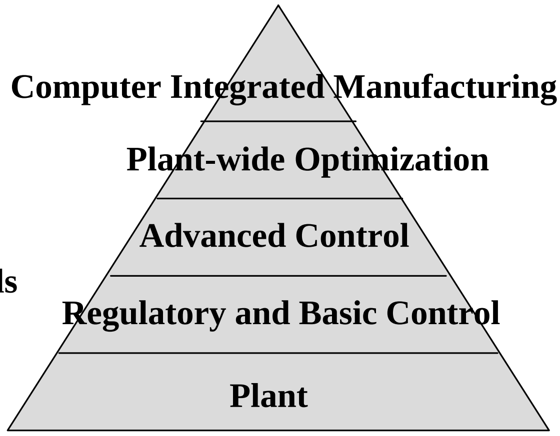
- **Emerging popularity of on-line optimization**
- **Process optimization and control are often conflicting objectives**
  - **Optimization pushes the process to the boundary of constraints.**
  - **Quality of control determines how close one can push the process to the boundary.**
- **Implications for process control**
  - **High performance control is needed to realize on-line optimization.**
  - **Constraint handling is a must.**
  - **The appropriate tradeoff between optimization and control is time-varying and is best handled within a single framework**

- **Synergy Between Optimization and Control**

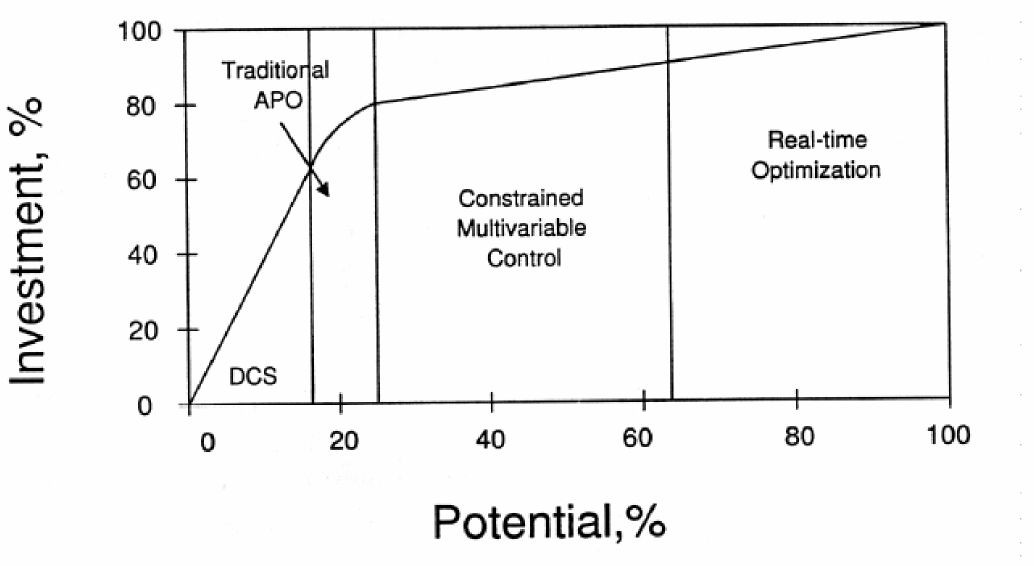




- **Control Hierarchy**
  - **Regulatory and basic control**
    - **PID control loops, cascade loops, independent actuators, etc.**
    - **The set point of each loop are given by advanced control**
    - **Fast sampling time and response (seconds or less)**
  - **Advanced control**
    - **Such as MPC**
    - **Manipulates the set points of the regulatory and basic controls**
    - **Higher-level set points are given by the plant-wide optimizer**
    - **Sampling time (seconds to minutes)**
  - **Plant-wide optimization**
    - **Calculate the optimum steady-states operating conditions based on the strategy from CIM**
    - **Sampling time (hours)**
  - **CIM (Computer Integrated Manufacturing)**
    - **Reflect corporate strategy and market condition**
    - **Production schedule**
    - **Sampling time (months)**



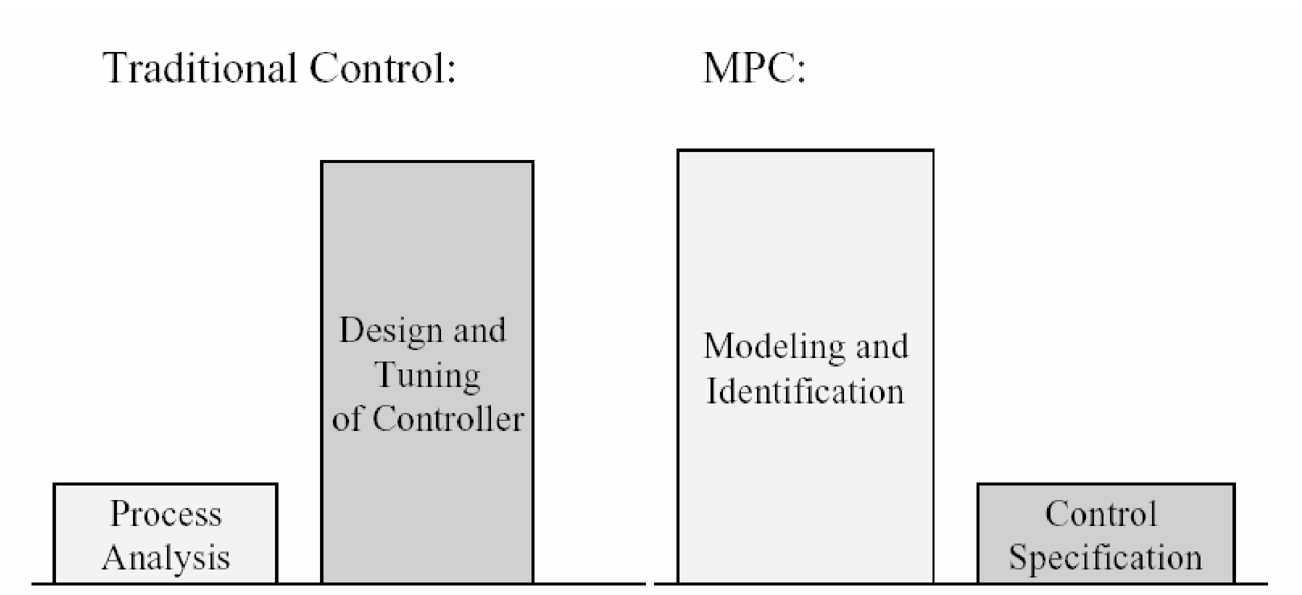
- **Return on Investment (ROI) for APC**



Importance of Modeling

- **Almost all models used in MPC are typically empirical models "identified" through plant tests rather than first-principles models.**
  - **Step responses, pulse responses from plant tests.**
  - **Transfer function models fitted to plant test data.**
- **Up to 80% of time and expense involved in designing and installing a MPC is attributed to modeling/system identification. → should be improved.**
- **Keep in mind that obtained models are imperfect (both in terms of structure and parameters).**
  - **Importance of feedback update of the model.**
  - **Penalize excessive input movements.**

- **Design effort**



Challenges

- **Efficient identification of control-relevant model**
- **Managing the sometimes exorbitant on-line computational load**
  - **Nonlinear models → Nonlinear Programs (NLP)**
  - **Hybrid system models (continuous dynamics + discrete events or switches, e.g., pressure swing adsorption)→ Mixed Integer Programs (MINLP)**
  - **Difficult to solve these reliably on-line for large-scale problems.**
- **How do we design model, estimator (of model parameters and state), and optimization algorithm as an integrated system - that are simultaneously optimized - rather than disparate components?**
- **Long-term maintenance of control system.**

Current Status on MPC

- **MPC is the established advanced multivariable control technique for the process industry. It is already an indispensable tool and its importance is continuing to grow.**
- **It can be formulated to perform some economic optimization and can also be interfaced with a larger-scale (e.g., plantwide) optimization scheme.**
- **Obtaining an accurate model and having reliable sensors for key parameters are key bottlenecks.**
- **A number of challenges remain to improve its use and performance.**

Process Models

- **Transfer function models**
  - Fixed order and structure
  - Parametric: few parameters to identify
  - Need very high order model for unusual behavior

- **Convolution models**
  - Continuous form

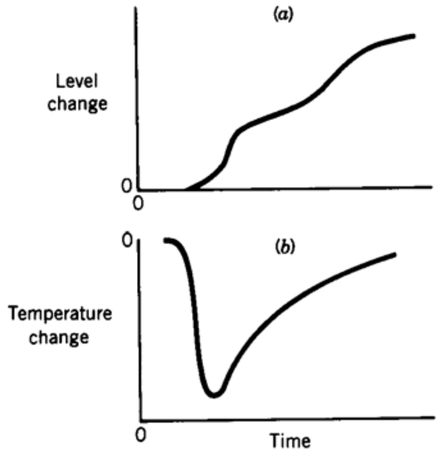
$$y(t) = \int_0^t h(\tau) u(t-\tau) d\tau$$

  - Discrete form

$$y(k) = \sum_{i=0}^{k} h(i) u(k-i)$$

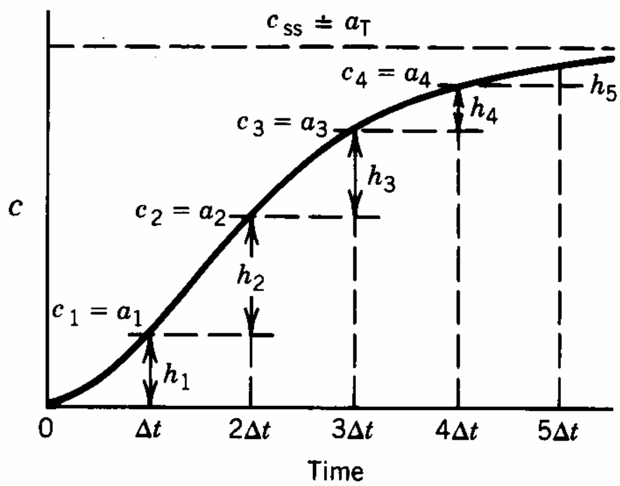
Impulse response

  - Many parameters, but easily obtained from the step or impulse response



Step Response Model

- **From open-loop step test**
  - Sampling time: Δt
  - Step response coefficients: a_i
  - Read the values of the unit step response



- **FSR model**
  - Finite step response (FSR)

$$y_k = a_k \ \ (u_k = 1, \forall k \geq 0)$$

  - Using superposition principle for arbitrary input changes

$$u_k = \Delta u_0 + \Delta u_1 + \cdots + \Delta u_k \ \text{ where } \Delta u_i = u_i - u_{i-1}$$

$$y_k = y_0 + y_k\big|_{\Delta u_0} + y_k\big|_{\Delta u_1} + \cdots + y_k\big|_{\Delta u_{k-1}}$$

$$= y_0 + a_k \Delta u_0 + a_{k-1} \Delta u_1 + \cdots + a_1 \Delta u_{k-1}$$

- **After $t = T\Delta t$, the step response reaches steady state at least 99%**

$$y_1 = y_0 + a_1 \Delta u_0$$
$$y_2 = y_0 + a_2 \Delta u_0 + a_1 \Delta u_1$$
$$y_3 = y_0 + a_3 \Delta u_0 + a_2 \Delta u_1 + a_1 \Delta u_2$$
$$\vdots$$
$$y_T = y_0 + a_T \Delta u_0 + a_{T-1} \Delta u_1 + \cdots + a_2 \Delta u_{T-2} + a_1 \Delta u_{T-1}$$
$$y_{T+1} = y_0 + a_T \Delta u_0 + a_T \Delta u_1 + a_{T-1} \Delta u_2 + \cdots + a_2 \Delta u_{T-1} + a_1 \Delta u_T$$
$$y_{T+2} = y_0 + a_T \Delta u_0 + a_T \Delta u_1 + a_T \Delta u_2 + a_{T-1} \Delta u_3 + \cdots + a_2 \Delta u_T + a_1 \Delta u_{T+1}$$
$$\vdots$$

$$\Rightarrow y_n = y_0 + \sum_{i=1}^{n} a_i \Delta u_{n-i} \quad (a_i = a_T, \forall i \geq T)$$ **(FSR Model)**

  - **If there is a delay, the FSR coefficients during the delay will be zero.**

Impulse Response Model

- **Impulse response coefficients**

$$h_i = a_i - a_{i-1} \quad (i = 1, 2, \cdots, T)$$
$$h_0 = 0$$

$$y_n = y_0 + \sum_{i=1}^{n} a_i \Delta u_{n+1-i} = y_0 + \sum_{i=1}^{n} a_i (u_{n+1-i} - u_{n-i})$$
$$= y_0 + (a_1 u_n - a_1 u_{n-1}) + (a_2 u_{n-1} - a_2 u_{n-2}) + \cdots + (a_n u_1 - a_n u_0)$$
$$= y_0 + a_1 u_n + (a_2 - a_1) u_{n-1} + \cdots + (a_n - a_{n-1}) u_1 - a_n \cancel{u_0}^{\,0}$$
$$= y_0 + (a_1 - \cancel{a_0}^{\,0}) u_n + (a_2 - a_1) u_{n-1} + \cdots + (a_n - a_{n-1}) u_1$$

$$\Rightarrow y_n = y_0 + \sum_{i=1}^{T} h_i u_{n-i} \quad (h_i = 0, \forall i \geq T)$$ **(FIR Model)**

Matrix Form of the Predictive Model

- **Horizons**
  - **Model horizon:** T **(number of model coefficients)**
  - **Control horizon:** U **(number of control moves)**
  - **Prediction horizon:** V **(number of predictions in the future)**

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_V \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ \vdots & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ a_V & a_{V-1} & a_{V-2} & & a_{V-U+1} \end{bmatrix} \begin{bmatrix} \Delta u_0 \\ \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{U-1} \end{bmatrix}$$

$$\mathbf{y} = \mathbf{A}\Delta\mathbf{u}$$

  - **A: Dynamic matrix**

Single-Step Prediction

- **From the FIR model**

$$\hat{y}_n = y_0 + \sum_{i=1}^{T} h_i u_{n-i} \qquad \hat{y}_{n+1} = y_0 + \sum_{i=1}^{T} h_i u_{n+1-i}$$

$$\Rightarrow \hat{y}_{n+1} = \hat{y}_n + \sum_{i=1}^{T} h_i \Delta u_{n+1-i}$$ **(Recursive prediction)**

- **Corrected prediction based on the measurement**
  - **Assume the error between the model prediction and the measurement will present in the future with same magnitude**

$$y^*_{n+1} - \hat{y}_{n+1} = y_n - \hat{y}_n \quad (y_n \text{ is the current measurement})$$

$$\Rightarrow y^*_{n+1} = \hat{y}_{n+1} + (y_n - \hat{y}_n) = y_n + \sum_{i=1}^{T} h_i \Delta u_{n+1-i}$$

Multi-Step Prediction

- **From the single-step prediction (*j*-step prediction)**

$$\hat{y}_{n+j} = \hat{y}_{n+j-1} + \sum_{i=1}^{T} h_i \Delta u_{n+j-i} \quad (j = 1, 2, \cdots, V)$$

$$y^*_{n+j} - \hat{y}_{n+j} = y^*_{n+j-1} - \hat{y}_{n+j-1} \quad (y_{n+j-1} \text{ is not available if } j>1)$$

$$\Rightarrow y^*_{n+j} = y^*_{n+j-1} + \sum_{i=1}^{T} h_i \Delta u_{n+j-i} \quad (j = 1, 2, \cdots, V)$$

- **Matrix form when V=U**

Dynamic Matrix, A

$$\begin{bmatrix} y^*_{n+1} \\ y^*_{n+2} \\ y^*_{n+3} \\ \vdots \\ y^*_{n+V} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ \vdots & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ a_V & a_{V-1} & a_{V-2} & & a_{V-U+1} \end{bmatrix} \begin{bmatrix} \Delta u_n \\ \Delta u_{n+1} \\ \Delta u_{n+2} \\ \vdots \\ \Delta u_{n+U-1} \end{bmatrix} + \begin{bmatrix} y_n + P_1 \\ y_n + P_2 \\ y_n + P_3 \\ \vdots \\ y_n + P_V \end{bmatrix}$$

where

$$P_i = \sum_{j=1}^{i} S_j \quad (i = 1, 2, \cdots, V)$$

$$S_j = \sum_{l=1}^{T} h_l \Delta u_{n+j-l} \quad (i = 1, 2, \cdots, V)$$

- S_j: **the incremental effect of the past (previously implemented) movements of input on the (*n*+*j*)-th future output prediction (where *n* is current time)**
- P_i: **the projection which includes future prediction of *y* based on all previously implemented input changes.**
- P_i **and** S_j **depend only on past input changes.**

- **If the past information is known, then the future input changes will affect the future outputs and the future outputs can be adjusted by carefully selecting the future inputs.**

- **Currently, n is current time and y_n is measured.**

$$y^*_{n+1} = y_n + \sum_{i=1}^{T} h_i \Delta u_{n+1-i} = h_1 \Delta u_n + y_n + \sum_{i=2}^{T} h_i \Delta u_{n+1-i} = a_1 \Delta u_n + y_n + \sum_{i=2}^{T} h_i \Delta u_{n+1-i}$$

$$y^*_{n+2} = y^*_{n+1} + \sum_{i=1}^{T} h_i \Delta u_{n+2-i} = (h_2 + h_1)\Delta u_n + h_1 \Delta u_{n+1} + \sum_{i=3}^{T} h_i \Delta u_{n+2-i} + y_n + \sum_{i=2}^{T} h_i \Delta u_{n+1-i}$$

$$y^*_{n+3} = y^*_{n+3} + \sum_{i=1}^{T} h_i \Delta u_{n+3-i}$$

$$= (h_2 + h_2 + h_1)\Delta u_n + (h_2 + h_1)\Delta u_{n+1} + h_1 \Delta u_{n+2} + \sum_{i=4}^{T} h_i \Delta u_{n+2-i} + y_n + \sum_{i=2}^{T} h_i \Delta u_{n+1-i} + \sum_{i=3}^{T} h_i \Delta u_{n+2-i}$$

⋮

$$y^*_{n+V} = y^*_{n+V-1} + \sum_{i=1}^{T} h_i \Delta u_{n+V-1-i} = a_V \Delta u_n + a_{V-1} \Delta u_{n+1} + \cdots + a_{V-U+1} \Delta u_{n+U-1}$$

$$+ y_n + \sum_{i=V+1}^{T} h_i \Delta u_{n+V-i} + \cdots + \sum_{i=3}^{T} h_i \Delta u_{n+2-i} + \sum_{i=2}^{T} h_i \Delta u_{n+1-i}$$

$$= a_V \Delta u_n + a_{V-1} \Delta u_{n+1} + \cdots + a_{V-U+1} \Delta u_{n+U-1} + y_n + \sum_{j=1}^{i} \sum_{i=j+1}^{T} h_i \Delta u_{n+j-i}$$

↑ **Depend on only future** ↑ **Depend on only past**

Controller Design Method (DMC)

- **Objective**
  - **Minimize errors between future set points and predictions**

$$\hat{\mathbf{E}} = \begin{bmatrix} r_{n+1} - y^*_{n+1} \\ r_{n+2} - y^*_{n+2} \\ \vdots \\ r_{n+V} - y^*_{n+V} \end{bmatrix} = \mathbf{r} - (\mathbf{A}\Delta\mathbf{u} + y_n \mathbf{e} - \mathbf{P}) = -\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}'$$

← **Closed-loop prediction error based only on current and future control action**

where

$$\hat{\mathbf{E}}' = \begin{bmatrix} r_{n+1} - y_n - P_1 \\ r_{n+2} - y_n - P_2 \\ \vdots \\ r_{n+V} - y_n - P_V \end{bmatrix}$$

← **Open-loop prediction error based only on past control action**

- **Solution**

$$-\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}' = \mathbf{0} \quad \Rightarrow \quad \Delta\mathbf{u} = (\mathbf{A}^*)^{-1}\hat{\mathbf{E}}'$$

← **Some inverse of A**

- **If $U=V$ and A is invertible,**

$$\Delta\mathbf{u} = \mathbf{A}^{-1}\hat{\mathbf{E}}'$$

It gives no steady-state offset since it has integral action.

- **If $U<V$ (A is not invertible),**

$$\Delta\mathbf{u} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\hat{\mathbf{E}}' = \mathbf{K}_c\hat{\mathbf{E}}'$$

A^+:Left pseudoinverse of A

$A^+A=I$: identity matrix

AA^+: *idempotent* matrix (BB=B)

- **Optimization concept**

$$\min(J = \hat{\mathbf{E}}^T\hat{\mathbf{E}}) = \min(-\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}')^T(-\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}')$$

$$\frac{\partial J}{\partial \Delta\mathbf{u}} = -2\mathbf{A}^T(-\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}') = 2(\mathbf{A}^T\mathbf{A}\Delta\mathbf{u} - \mathbf{A}^T\hat{\mathbf{E}}') = 0$$

$$\Rightarrow \Delta\mathbf{u} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\hat{\mathbf{E}}'$$

$$\min J = (\hat{\mathbf{E}}^T\mathbf{W}_1\hat{\mathbf{E}} + \Delta\mathbf{u}^T\mathbf{W}_2\Delta\mathbf{u})$$

$$\frac{\partial J}{\partial \Delta\mathbf{u}} = -2\mathbf{A}^T\mathbf{W}_1(-\mathbf{A}\Delta\mathbf{u} + \hat{\mathbf{E}}') + 2\mathbf{W}_2\Delta\mathbf{u} = 2((\mathbf{A}^T\mathbf{W}_1\mathbf{A} + \mathbf{W}_2)\Delta\mathbf{u} - \mathbf{A}^T\mathbf{W}_1\hat{\mathbf{E}}') = 0$$

$$\Rightarrow \Delta\mathbf{u} = (\mathbf{A}^T\mathbf{W}_1\mathbf{A} + \mathbf{W}_2)\mathbf{A}^T\mathbf{W}_1\hat{\mathbf{E}}'$$

- **Adjustable parameters of MPC (Tuning parameters)**
  - **Weighting matrices**
    - **If W_1>>W_2, the most important objective is to minimize error of the process outputs and inputs will move quite freely.**
    - **If W_1<<W_2, the most important objective is to minimize the input movements and controller cares much less the errors. (almost no control)**
    - **Otherwise, it depends on the relative size of the weighting matrices.**
      - **If W_1>W_2, aggressive action will be taken to reduce the error.**
      - **If W_1<W_2, conservative action will be taken to reduce the input movements while reduce the error if the action is not too aggressive.**
    - **The W_2 is called *input penalty* or *input move suppression factor*.**
    - **Typically, use W_1=I and W_2=f^2I and adjust f.**
    - **If a different weighting for outputs or inputs is required, use diagonal matrix as the weighting matrix.**

– **Horizons**

- **Model horizon (*T*)**
  - **Select *T* such that** $T\Delta t \geq$ (open-loop settling time)
  - ***T* is typically 20 to 70.**
- **Prediction horizon (*V*)**
  - **Increasing *V* results in more conservative control action, a stabilizing effect, and more computational burden.**
  - **An important tuning parameter**
- **Control horizon (*U*)**
  - **Suitable first guess is to choose *U* so that** $U\Delta t \cong t_{60}$
  - **The larger the value of *U* is, the more computation time is required.**
  - **Too large a value of *U* results in excessive control action**
  - **Smaller value of U leads to a robust controller that is relatively insensitive to model error.**

MIMO Extension

- **2x2 case**

$$\widehat{\mathbf{E}} = -\mathbf{A}\Delta\mathbf{u} + \widehat{\mathbf{E}}'$$

where

$$\widehat{\mathbf{E}} = [\widehat{\mathbf{E}}_1;\ \widehat{\mathbf{E}}_2] \qquad \Delta\mathbf{u} = [\Delta\mathbf{u}_1;\ \Delta\mathbf{u}_2]$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

- **General case**
  - **Extend the vectors and matrices in the same manner.**
  - **If the MPC is formulated in a different form such as state-space model, different form of MIMO extension is more convenient.**

Constraints Handling

- **Formulate and solve the MPC in an optimization framework**

$$\min J = (\hat{\mathbf{E}}^T \mathbf{W}_1 \hat{\mathbf{E}} + \Delta\mathbf{u}^T \mathbf{W}_2 \Delta\mathbf{u})$$

$$\text{subject to } \mathbf{u}^L \le \mathbf{u} \le \mathbf{u}^U$$

$$\mathbf{y}^L \le \mathbf{y} \le \mathbf{y}^U$$

and other constraints

- **Solve this optimization problem in QP**
  - **DMC by DMCC used LP**

Model Algorithmic Control (MAC)

- **Process model:**
  - **based on** u **not** Δu
- **Set point:**
  - **First-order approach to set point**

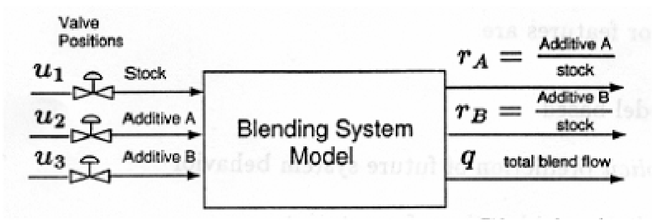
$$r^*_{k+i} = \alpha y_{k+i-1} + (1-\alpha) r_{k+i}$$

  - **Speed of response is determined by** α **(tuning parameter)**
- **Tuning parameters**
  - **Speed of desired response**
  - $U=V$ **(fixed, not used as tuning parameters)**
  - V **is chosen so that** $V\Delta t \approx$ (open-loop settling time)
  - **Time varying weight:** $J = \sum_{i=1}^{V} w(i) e(k+i)^2$
- **Solution is obtained using QP**

Comments on MPC

- **Implementation**
  - **Update the prediction model based on the current measurement.**
  - **Calculate *U* moves from the optimization and implement the first input moves and throw out the rest.**
- **The MPC is minimizing the error between the set point and predicted output.**
  - **In the prediction, the measurement is incorporated and it works as a feedback.**
  - **No steady-state offset: integrator in the control law**
- **Disturbance Model can be added**
  - **Known measured disturbance can be incorporated by adding disturbance model in the same manner.**

- **Example 1: Blending control system**



Objectives:

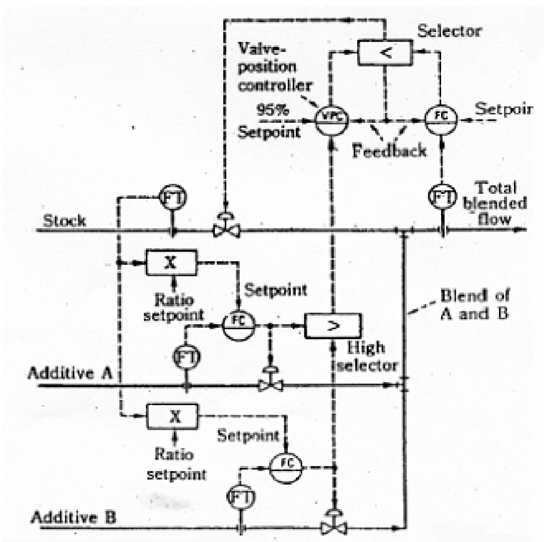
-- Control the composition of A and B

-- Control total flow if possible

Constraints:

-- Flow rates are limited

Classical solution



MPC solution

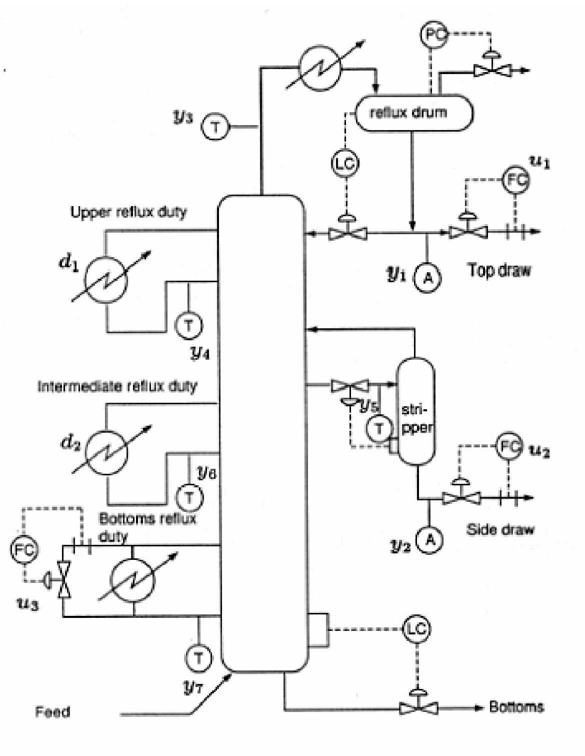
$$\min \sum_{i=1}^{V} (r_A(k+i|k) - r_A^*)^2 + (r_B(k+i|k) - r_B^*)^2 + w(q(k+i|k) - q^*)^2$$

$$\text{subject to } u_i^L \le u_i\,(j) \le u_i^L \quad (i = 1, 2, 3)$$

$$(j = k, \cdots, k+U-1)$$

$$w \ll 1$$

• Example 2: Heavy Oil Fractionator



- Keep $y_7 \ge T_{min}$
- Control the two compositions y_1 and y_2
- Minmize u_3 to maximize the heat recovery.

$$\min \sum_{i=1}^{V} (y_1(k+i|k) - y_1^*)^2 + (y_2(k+i|k) - y_2^*)^2 + w_1(u_3)^2$$

$$\text{subject to } u_i^L \le u_i\,(j) \le u_i^L \;\; (i = 1,2,3)$$

$$(j = k, \cdots, k+U\text{ -}1)$$

$$y_7 \ge T_{min}$$

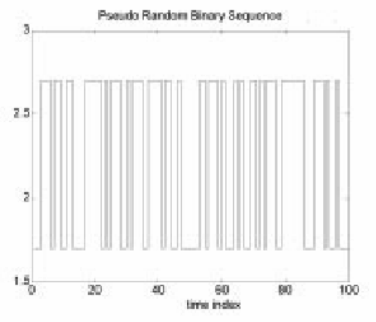
Identification of Models

- **FSR or FIR models: use step or pulse test**
  - **Assume operation at steady state**
  - **Make change in input Δu (or δu)**
    - **If Δu is too small, output change may not noticeable**
    - **If Δu is too large, linearity may not hold**
  - **Measure output at regular intervals Δt**
    - **The Δt should be chosen so that T is 20-70,typically 40.**
  - **Perform multiple experiments and average them and additional experiments for verification**
  - **High frequency information may not be accurate for step test.**
  - **Ideal pulse is hard to implement.**

• Least Squares Identification

– **Get the output using PRBS (Pseudo Random Binary Signal)**

$\mathbf{u} = [u_1 \ u_2 \ \cdots \ u_M] \qquad \mathbf{y} = [y_1 \ y_2 \ \cdots \ y_M]$



– **Get the FIR model**

$\tilde{y}_k = h_1 u_{k-1} + h_2 u_{k-2} + \cdots + h_N u_{k-N}$

– **Minimize the error between measurements and output,** $d_k = y_k - \tilde{y}_k$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} u_0 & u_{-1} & \cdots & u_{1-N} \\ u_1 & u_0 & \cdots & u_{2-N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{M-1} & u_{M-2} & \cdots & u_{M-N} \end{bmatrix} \begin{bmatrix} h_1 \\ h_1 \\ \vdots \\ h_N \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_M \end{bmatrix} \qquad \mathbf{d} = \mathbf{y} - \mathbf{U}\mathbf{h}$$

$$\min_{\mathbf{h}} \mathbf{d}^T \mathbf{d} = \min_{\mathbf{h}} (\mathbf{y} - \mathbf{U}\mathbf{h})^T (\mathbf{y} - \mathbf{U}\mathbf{h}) \Rightarrow \mathbf{h} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{y}$$

• Discussions

– **Random input testing, if appropriately designed, gives better models than the step or pulse testing does since it can equally excite low to high frequency dynamics of the process.**

– **If U^TU is singular, the inverse doesn't exist and identification fails. (Need persistent excitation condition)**

– **When the number of coefficients is large, U^TU can be easily singular (or nearly singular). To avoid the numerical, a regularization term is added the the cost function. (ridge regression)**

$$\min_{\mathbf{h}} [(\mathbf{y} - \mathbf{U}\mathbf{h})^T (\mathbf{y} - \mathbf{U}\mathbf{h}) + \alpha \mathbf{h}^T \mathbf{h}] \Rightarrow \mathbf{h} = (\mathbf{U}^T \mathbf{U} + \alpha \mathbf{I})^{-1} \mathbf{U}^T \mathbf{y}$$

Data Treatments

- **The data need to be processed before they are used in identification.**

- **Spike/Outlier Removal**
  - **Check plots of data and remove obvious outliers (e.g., that are impossible with respect to surrounding data points). Fill in by interpolation.**
  - **After modeling, plot of actual vs. predicted output (using measured input and modeling equations) may suggest additional outliers. Remove and redo modeling, if necessary.**
  - **But don't remove data unless there is a clear justification.**

- **Bias Removal and Normalization**
  - **Compute the data average and subtract it to create deviation variables, i.e.,**

$$\hat{y}_k = (y_k - y_{ref})/c_y \text{ where } y_{ref} = \sum_{i=1}^{M} y_i / M$$

$$\hat{u}_k = (u_k - u_{ref})/c_u \text{ where } u_{ref} = \sum_{i=1}^{M} u_i / M$$

  - **Use the given steady-state values of the variables instead to compute the deviation variables, i.e.,**

$$\hat{y}_k = (y_k - y_{ss})/c_y \text{ and } \hat{u}_k = (u_k - u_{ss})/c_u$$

where y_{ss} and u_{ss} represent a priori given steady-state values of the process output and input respectively.

  - **The input/output data can be biased by the nonzero steady state and also by load disturbance effects. To remove the (time-varying) bias, differencing can be performed for the input/output data.**

$$\Delta y_k = (y_k - y_{k-1})/c_v \text{ and } \Delta u_k = (u_k - u_{k-1})/c_u$$

$$\Rightarrow \text{Identification for } \Delta y_k \text{ and } \Delta u_k$$

  - **In all cases, the process data are conditioned by scaling before using in identification.**

- **Prefiltering**
  - **If the data contain too much frequency components over an undesired range and/or if we want to obtain a model that fits well the data over a certain frequency range, data prefiltering (via digital filters) can be done.**